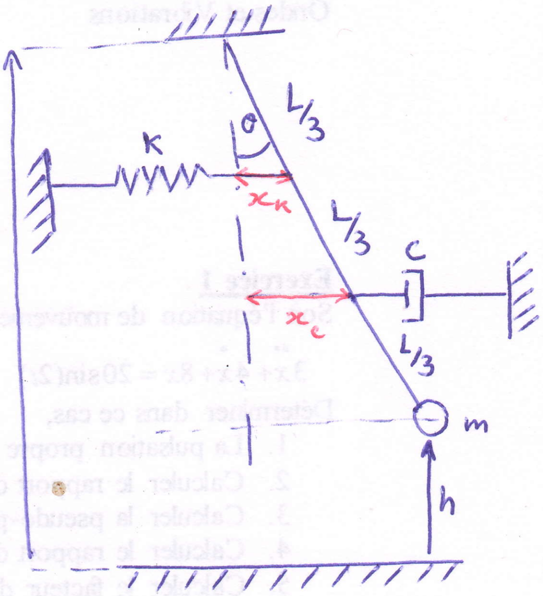
# Solution TD N°2




## Exo1

1) L'eq du mvt $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) - \frac{\partial L}{\partial \theta} = -\frac{\partial D}{\partial \dot\theta}$

$$T = \frac{1}{2} J . \dot\theta^2 = \frac{1}{2} m L^2 \dot\theta^2$$

$$V = \frac{1}{2} K x_k^2 + m g h = \frac{1}{2} K \left(\frac{L}{3}\sin\theta\right)^2 + mg\left(H - L\cos\theta\right)$$

$$= \frac{1}{18} . K . L^2 \sin^2\theta + mgH - mg . L\cos\theta$$

$$D = \frac{1}{2} c \underset{\uparrow}{\dot x_c}^2 = \frac{1}{2} c \left(\frac{2L}{3} . \dot\theta \cos\theta\right)^2$$

$x_c = \frac{2L}{3} . \sin\theta$

$$L = T - V$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot\theta}\right) = m . L^2 . \ddot\theta \quad ; \quad \frac{\partial L}{\partial \theta} = -\frac{KL^2}{9}\cos\theta \sin\theta - mg . L\sin\theta$$

$$\frac{\partial D}{\partial \dot\theta} = \frac{4L^2}{9} . c . \cos^2\theta . \dot\theta$$

Eq du mvt :

$$m L^2 \ddot\theta + \frac{KL^2}{9} \cancelto{1}{\cos\theta} \cancelto{\theta}{\sin\theta} + mgL \cancelto{\theta}{\sin\theta} + \frac{4L^2}{9} c \cancelto{1}{\cos^2\theta} . \dot\theta = 0$$

$$\boxed{m L^2 \ddot\theta + \frac{4}{9} c . L^2 . \dot\theta + \left(\frac{KL^2}{9} + mg . L\right)\theta = 0}$$

On a : $m = 0{,}5$ Kg ; $K = 4\,N.m^{-1}$ ; $C = 8\,Kg.s^{-1}$ ; $L = 0{,}5\,m$ ; $g = 10\,m.s^{-2}$

$$0{,}125\,\ddot\theta + 0{,}8\,\dot\theta + 2{,}61\,\theta = 0$$

$$\boxed{\ddot\theta + \underbrace{6.4}_{2\alpha\omega_0}\,\dot\theta + \underbrace{20{,}88}_{\omega_0^2}\,\theta = 0}$$

2) Pulsation propre

$$\boxed{\omega_0 = \sqrt{20{,}88} = 4{,}57\ rd/s}$$

3) Rapport d'amortissement $\alpha$

$2\alpha\omega_0 = 6{,}4 \rightarrow \alpha = \frac{6{,}4}{2\omega_0} = \frac{6{,}4}{9.14} = 0{,}7$

$\boxed{\alpha = 0{,}7 < 1}$ $\Rightarrow$ Amortissement faible (Sous critique)

4) Pseudo-pulsation $\omega_a$

$$\omega_a = \omega_0 \sqrt{1-\alpha^2} = 4,57\sqrt{1-(0,7)^2} = 3,26$$

$$\boxed{\omega_a = 3,26 \ rd/s}$$

5) Pseudo-période Ta

$$\boxed{T_a = \frac{2\pi}{\omega_a} = 1,93 \ s}$$

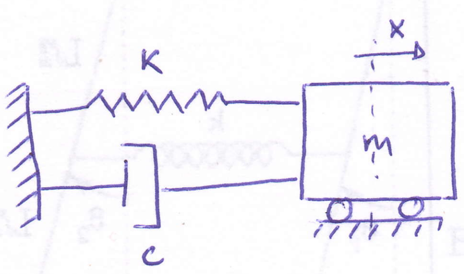
6) Solution générale

$$\theta(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} \quad \left( r_{1,2} = -\alpha\omega_0 \pm \omega_0\sqrt{\alpha^2-1} \right)$$

$A_1$ et $A_2$ sont déterminés à partir des conditions initiales $\theta(t=0)$, $\dot{\theta}(t=0)$

Ex 02

Syst masse-ressort-amortisseur




Eq du mvt : $m\ddot{x} + c\dot{x} + Kx = 0$

$$\ddot{x} + \underbrace{\frac{c}{m}}_{2\alpha\omega_0}\dot{x} + \underbrace{\frac{K}{m}}_{\omega_0^2} x = 0$$

On a : $m = 150 g = 0,15 \ Kg$ ; $c = 0,6 \ Kg/s$ ; $K = 3,8 \ N/m$.

$$\boxed{\ddot{x} + 4\dot{x} + 25,3x = 0}$$

1) Pulsation propre $\omega_0$

$$\omega_0 = \sqrt{\frac{K}{m}} = \sqrt{25,3} = 5,03 \ rd/s$$

2) Type d'amortissement

$$2\alpha\omega_0 = \frac{c}{m} = 4 \Rightarrow \alpha = \frac{4}{2 \cdot 5,03} \simeq 0,4 < 1$$ : Amortissement faible

3) Solution de l'eq du mvt :

conditions initiales $x(t=0) = 0$ , $\dot{x}(t=0) = 0,25 \ m/s$

$$x(t) = A_1 e^{r_1 t} + A_2 e^{r_2 t} = e^{-\alpha\omega_0 t}(A\cos\omega_a t + B\sin\omega_a t) = \underbrace{a}_{\text{Amplitude}} e^{-\alpha\omega_0 t}\sin(\omega_a t + \underbrace{\varphi}_{\text{phase}})$$

- $x(t=0) = a e^0 \sin\varphi = 0 \Rightarrow a \neq 0$ Donc $\sin\varphi = 0 \Rightarrow \boxed{\varphi = 0 \text{ ou } \varphi = \pi}$
- $\dot{x}(t) = -a\alpha\omega_0 e^{-\alpha\omega_0 t}\sin(\omega_a t) + a\omega_a e^{-\alpha\omega_0 t}\cos(\omega_a \cdot t)$

$$\dot{x}(t=0) = a\omega_a = 0,25 \Rightarrow a = \frac{0,25}{\omega_a} = \frac{0,25}{\omega_0\sqrt{1-\alpha^2}} = 0,054 \ m \Rightarrow \boxed{a = 0,054 \ m}$$

$$\boxed{x(t) = 0,054 e^{-2t}\sin(4,6 \cdot t)} \rightarrow \text{Solution } x(t)$$