

Corrigé de la fiche des travaux dirigés N°1

Solution 01 :

1. Les températures et les pressions aux différents points du cycle

- Pt (1) : $T_1 = 65^\circ\text{C} = 338\text{ K}$ et $P_1 = 130\text{ kPa}$.

- Pt (2) :

1 → 2: Transformation isentropique (adiabatique réversible)

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = T_1 (\varepsilon_c)^{\gamma-1} = 338 * 19^{0.4}$$

$$T_2 = 1097.53\text{ K}$$

$$P_1 V_1^\gamma = P_2 V_2^\gamma$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^\gamma = P_1 (\varepsilon_c)^\gamma = 130 * 19^{1.4}$$

$$P_2 = 8020.44\text{ kPa}$$

- Pt (3) :

2 → 3: Transformation Isobare

$$P_3 = P_2$$

$T_3 = ?$

$$Q_{in} = m_m * C_p (T_3 - T_2) \dots \dots \dots (1)$$

$$\eta_c = \frac{Q_{in}}{m_f Pci} \Leftrightarrow Q_{in} = \eta_c * m_f Pci \dots \dots \dots (2)$$

$$(1) = (2) \Leftrightarrow \eta_c * m_f * Pci = m_m * C_p (T_3 - T_2) \dots \dots \dots (3)$$

$$m_m = m_f + m_a$$

$$AF = \frac{m_a}{m_f} \Leftrightarrow m_a = AF * m_f$$

$$m_m = m_f (1 + AF)$$

$$(3) \Leftrightarrow T_3 = T_2 + \frac{\eta_c * Pci}{(1 + AF) * C_p} = 1097.53 + \frac{0.98 * 42500}{1.004 * (1 + 18.125)}$$

$$T_3 = 3266.63\text{ K}$$

- Pt (4) :

3 → 4: Transformation isentropique (adiabatique réversible)

$$T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}$$

$$T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{\gamma-1}$$

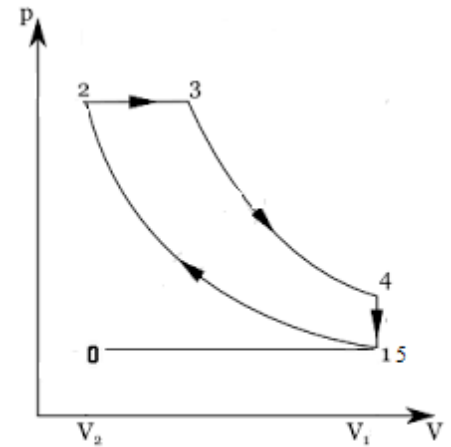
$$V_3 = ? \text{ et } V_4 = ?$$

$$V_4 = V_1 = \frac{R * T_1}{P_1} = \frac{0.287 * 338}{130}$$

$$V_4 = 0.7462\text{ m}^3/\text{kg}$$

$$V_3 = \frac{R * T_3}{P_3} = \frac{0.287 * 3266.03}{8020.44}$$

$$V_3 = 0.1168\text{ m}^3/\text{kg}$$



Cycle Diesel

$$T_4 = T_3 \left(\frac{V_3}{V_4} \right)^{\gamma-1} = 3266.63 \left(\frac{0.1168}{0.7462} \right)^{\gamma-1}$$

$$T_4 = 1555.73 \text{ K}$$

$$P_3 V_3^\gamma = P_4 V_4^\gamma$$

$$P_4 = P_3 \left(\frac{V_3}{V_4} \right)^\gamma = 8020.44 \left(\frac{0.1168}{0.7462} \right)^{1.4}$$

$$P_4 = 597.889 \text{ kPa}$$

2. La quantité de chaleur ajoutée durant la combustion

$$Q_{in} = Q_{23} = C_p (T_3 - T_2)$$

$$Q_{in} = 1.004 * (3266.63 - 1097.53)$$

$$Q_{in} = 2178.30 \frac{\text{kJ}}{\text{kg}}$$

3. La quantité de chaleur à la détente

$$Q_{out} = Q_{41} = C_v (T_1 - T_4) = \frac{C_p}{\gamma} (T_1 - T_4)$$

$$Q_{out} = \frac{1.004}{1.4} * (338 - 1555.73)$$

$$Q_{out} = -873.28 \frac{\text{kJ}}{\text{kg}}$$

4. Le rendement thermique indiqué

$$\eta_{th} = \frac{W_{net}}{Q_{in}} = 1 - \left| \frac{Q_{out}}{Q_{in}} \right| \Leftrightarrow W_{net} = \eta_{th} * Q_{in}$$

$$\eta_{th} = 1 - \left| \frac{873.28}{2178.30} \right|$$

$$\eta_{th} = 60\%$$

Solution 02 :

Partie 1 :

1. Calcul des masses constituantes du mélange dans le cylindre ;

La masse du mélange (m_m) dans le cylindre est constituée de l'air (m_a), du fuel (m_f) et des gaz résiduels (m_r).

$$m_r \rightarrow 0.04 * m_m$$

$$m_m = \frac{P_1 V_1}{RT_1} = 0.000740 kg$$

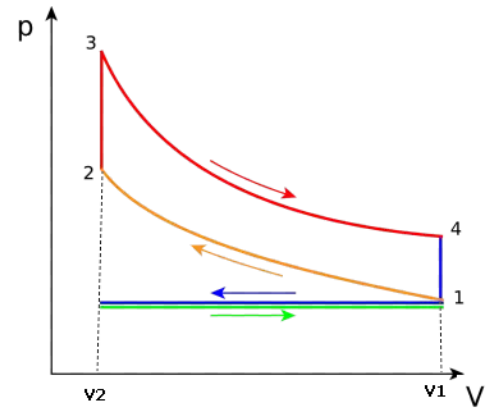
$$m_r = 0.04 * m_m = 0.0000296 kg$$

$$m_a + m_f \rightarrow 0.96 * m_m$$

$$AF = 15 \Leftrightarrow \frac{m_a}{m_f} = 15 \Leftrightarrow m_a = 15m_f$$

$$0.96m_m = m_a + m_f = 15m_f + m_f \Leftrightarrow m_f = 0.0000440 kg$$

$$m_a = 0.000666 kg$$



2. Calcul des paramètres thermodynamiques (P_i, V_i, T_i) de cycle

$$V_d = \frac{2.5}{4} = 0.625 = 0.000625 m^3$$

$$\varepsilon = \frac{V_{max}}{V_{min}} = \frac{V_1}{V_2} = \frac{V_{cc} + V_d}{V_{cc}} = 8.6$$

$$V_{cc} + V_d = 8.6V_{cc} \Leftrightarrow V_{cc} = \frac{V_d}{7.6} = 0.0000822 m^3$$

$$Pt (1) \left\{ \begin{array}{l} T_1 = 60^\circ C = 333K \\ P_1 = 100kPa \\ V_1 = V_d + V_{cc} = 0.000625 + 0.0000822 \\ V_1 = 0.0007072 m^3 \end{array} \right.$$

Pt (2): 1 $\rightarrow$ 2: transformation adiabatique

$$\left\{ \begin{array}{l} T_2 = T_1 \varepsilon^{\gamma-1} = 707.16K \\ P_2 = P_1 \varepsilon^\gamma = 1826.30kPa \\ V_2 = \varepsilon * V_1 \\ V_2 = 0.0000822 m^3 \end{array} \right.$$

Pt (3): 2 $\rightarrow$ 3: transformation Isochore

$$\left\{ \begin{array}{l} P_3 = P_2 (T_3/T_2) = 10111kPa \\ V_3 = V_2 = 0.0000822 m^3 \\ Q_{23} = m_m C_v (T_3 - T_2) \\ Q_{23} = m_f * Pci * \eta_c = 1.949kJ \\ T_3 = \frac{Q_{23}}{m_m C_v} + T_2 = 3915K \end{array} \right.$$

Pt (4): 3 $\rightarrow$ 4: transformation adiabatique

$$\left\{ \begin{array}{l} \frac{T_4}{T_3} = \left(\frac{V_3}{V_4} \right)^{\gamma-1} \Rightarrow T_4 = T_3 \left(\frac{1}{\varepsilon} \right)^{\gamma-1} \Rightarrow T_4 = 1844K \\ \frac{P_4}{P_3} = \left(\frac{V_3}{V_4} \right)^\gamma \Rightarrow P_4 = \left(\frac{1}{\varepsilon} \right)^\gamma = 554kPa \\ V_4 = V_1 = 0.000707 m^3 \end{array} \right.$$

3. Calcul du rendement thermique indiqué :

$$\eta_{th} = \left| \frac{W_{net}}{Q_{23}} \right|$$

$$W_{43} = \frac{m_m R}{1 - \gamma} (T_4 - T_3) = \frac{0.000740 * 0.287}{1 - 1.35} (1843 - 3915)$$

$$W_{43} = 1.257kJ$$

$$W_{12} = \frac{m_m R}{1 - \gamma} (T_2 - T_1) = \frac{0.000740 * 0.287}{1 - 1.35} (707 - 333)$$

$$W_{43} = -0.227kJ$$

$$W_{net} = 1.03kJ$$

$$\eta_{th} = \left| \frac{W_{net}}{Q_{23}} \right| = 52.84\%$$

Partie 2 :

$$V_h = \frac{2.5}{4} = 0.625 = 0.000625 m^3$$

$$\varepsilon_d = \frac{V_h + V_{cc}}{V_{cc}} = \frac{V_4}{V_3} \Rightarrow V_{cc} = 0.000069m^3$$

$$V_{cc} = V_2 = V_3$$

$$V_1 = V_4 = V_5 = V_{cc} + V_h = 0.000625 + 0.000069 = 0.000694m^3$$

1. Températures et pressions

$$Pt (7) \left\{ \begin{array}{l} T_7 = 60^\circ C = 333K \\ P_7 = P_5 = 160kPa \\ \varepsilon_c = \frac{V_7}{V_2} \Rightarrow V_7 = V_2 \varepsilon_c = 0.000552m^3 \end{array} \right.$$

$$Pt (1) \left\{ \begin{array}{l} T_1 = T_7 \left(\frac{V_7}{V_1} \right)^{k-1} = 307K \\ P_1 = P_7 \left(\frac{V_7}{V_1} \right)^k = 117.46kPa \\ V_1 = 0.000694m^3 \end{array} \right.$$

$$Pt (2) \left\{ \begin{array}{l} T_2 = T_7 \varepsilon_c^{k-1} = 689K \\ P_2 = P_7 \varepsilon_c^k = 2650K \\ V_2 = V_c = 0.000069m^3 \end{array} \right.$$

$$m_m = m_1 = \frac{P_1 V_1}{RT_1} = 0.000922kg$$

$$AF = 15 \Leftrightarrow \frac{m_a}{m_f} = 15 \Leftrightarrow m_a = 15m_f$$

$$m_a + m_f \rightarrow 0.96 * m_m$$

$$m_r \rightarrow 0.04 * m_m$$

$$0.96m_m = m_a + m_f = 15m_f + m_f \Leftrightarrow m_f = \frac{0.96 * 0.000922}{16} = 0.000055kg$$

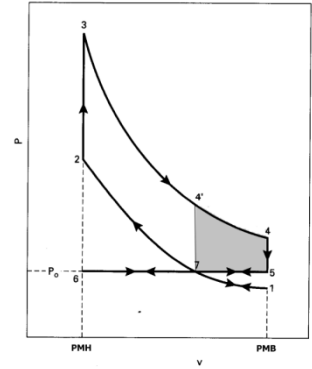


Fig. 8: diagramme p-V du cycle de Miller

$$m_a = 0.00083kg$$

$$m_r = 0.04 * m_m = 0.000036kg$$

$$Pt (3) \left\{ \begin{array}{l} P_3 = P_2(T_3/T_2) = 15031kpa \\ V_3 = V_2 = 0.000069m^3 \\ Q_{23} = m_m C_v (T_3 - T_2) \\ Q_{23} = Q_{in} = m_f * Pci * \eta_c = 2.437kJ \\ T_3 = \frac{Q_{23}}{m_m C_v} + T_2 = 3908K \end{array} \right.$$

Donnée : $Pci = 44300 \frac{kJ}{kg}$ et $\eta_c = 100\%$

$$Pt (4) \left\{ \begin{array}{l} \frac{T_4}{T_3} = \left(\frac{V_3}{V_4}\right)^{k-1} \Rightarrow T_4 = T_3 \left(\frac{1}{\varepsilon_d}\right)^{k-1} \Rightarrow T_4 = 1746K \\ \frac{P_4}{P_3} = \left(\frac{V_3}{V_4}\right)^k \Rightarrow P_4 = 671kPa \\ V_4 = V_5 = V_d + V_c = 0.000694m^3 \end{array} \right.$$

2. Rendement thermique :

$$\eta_t = \left| \frac{W_{net}}{Q_{in}} \right|$$

$$W_{net} = W_{12} + W_{34}$$

$$W_{12} = m_m R(T_2 - T_1)/(1 - \gamma) = -0.288kJ$$

$$W_{34} = m_m R(T_4 - T_3)/(1 - \gamma) = 1.635kJ$$

$$W_{net} = 1.366kJ$$

$$\eta_t = 56\%$$

3. La température d'échappement

$$T_{ech} = T_4 \left(\frac{P_{ext}}{P_4}\right)^{\frac{k-1}{k}} = 1746 \left(\frac{100}{671}\right)^{\frac{0.35}{1.35}} = 2208 K$$

	Cycle Miller	Cycle Otto
Température au début de la combustion :	689K	707K
Pression au début de la combustion	2650kPa	1826kPa
Température maximale	3908K	3915K
Pression maximale	15031 kPa	10111kPa
Température d'échappement	1066K	1183K
Rendement thermique	56%	52.84%

Solution 03 :

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = T_1 \varepsilon_c^{\gamma-1} = 300(8.5)^{0.4} = 706.136K$$

$$Q_{23} = C_v(T_3 - T_2)$$

$$T_3 = \frac{1400}{0.718} + 706.136 = 2656K$$

$$T_5 = T_3 \left(\frac{P_3}{P_5} \right)^{\frac{1-\gamma}{\gamma}}$$

$$P_5 = P_1$$

$$P_3 = T_3 \left(\frac{P_2}{T_2} \right) = 2656 \frac{16005.766}{706.136} = 60202.73kPa$$

$$P_2 = P_1 \left(\frac{V_1}{V_2} \right)^{\gamma} = P_1 \varepsilon_c^{\gamma} = 800(8.5)^{1.4} = 16005.766kPa$$

$$T_5 = 2656 \left(\frac{60202.73}{800} \right)^{\frac{-0.4}{1.4}} = 772.82K$$

La chaleur perdue Q_{51}

$$Q_{51} = C_p(T_1 - T_5) = 1.005(300 - 772.82) = -475.18kJ/kg$$

Le travail du cycle

$$W_{net} = W_{12} + W_{35}$$

Premier principe de la thermodynamique : $\Delta U = 0 \Leftrightarrow W_{12} + W_{35} = Q_{51} + Q_{23}$

$$W_{net} = Q_{51} + Q_{23} = -475.18 + 1400 = 924.82kJ/kg$$

Le rendement du cycle

$$\eta_{th} = \left| \frac{W_{net}}{Q_{23}} \right| = 66.05\%$$