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Numbers, Symbols and Equations

1) Numbers

A number is a mathematical object used to count, measure, and label. The cardinal numbers (one, two, three, etc.) are adjectives referring to quantity, and the ordinal numbers (first, second, third, etc.) refer to distribution.

Number	**Cardinal**	**Ordinal**
1	one	first
2	two	second
3	three	third
4	four	fourth
5	five	fifth
6	six	sixth
7	seven	seventh

Number	Cardinal	Ordinal
8	eight	eighth
9	nine	ninth
10	ten	tenth
11	eleven	eleventh
12	twelve	twelfth
13	thirteen	thirteenth
14	fourteen	fourteenth
15	fifteen	fifteenth
16	sixteen	sixteenth

Number	**Cardinal**	**Ordinal**
17	seventeen	seventeenth
18	eighteen	eighteenth
19	nineteen	nineteenth
20	twenty	twentieth
21	twenty-one	twenty-first
22	twenty-two	twenty-second
23	twenty-three	twenty-third
24	twenty-four	twenty-fourth
25	twenty-five	twenty-fifth

Number	**Cardinal**	**Ordinal**
26	twenty-six	twenty-sixth
27	twenty-seven	twenty-seventh
28	twenty-eight	twenty-eighth
29	twenty-nine	twenty-ninth
30	thirty	thirtieth
40	forty	fortieth
50	fifty	fiftieth
60	sixty	sixtieth
70	seventy	seventieth

Number	**Cardinal**	**Ordinal**
80	eighty	eightieth
90	ninety	ninetieth
100	one hundred	hundredth
500	five hundred	five hundredth
1,000	one thousand	thousandth
1,500	one thousand five hundred, or fifteen hundred	one thousand five hundredth
100,000	one hundred thousand	hundred thousandth
1,000,000	one million	millionth

Examples

- There are **twenty-five** people in the room.
- **Six hundred thousand** people were left homeless after the earthquake.

Read decimals aloud in English by pronouncing the decimal point as "point", then read each digit individually. Money is not read this way.

Written	Said
0.5	point five
0.25	point two five
0.73	point seven three
0.05	point zero five
0.6529	point six five two nine
2.95	two point nine five

Read fractions using the cardinal number for the numerator and the ordinal number for the denominator, making the ordinal number plural if the numerator is larger than 1. This applies to all numbers except for the number 2, which is read "half" when it is the denominator, and "halves" if there is more than one.

Written	Said
1/3	one third

Written	Said
3/4	three fourths
5/6	five sixths
1/2	one half
3/2	three halves

How to say 0

There are several ways to pronounce the number 0, used in different contexts. Unfortunately, usage varies between different English-speaking countries. These pronunciations apply to American English.

Pronunciation	Usage
zero	Used to read the number by itself, in reading decimals, percentages, and phone numbers, and in some fixed expressions.
o (the letter name)	Used to read years, addresses, times and temperatures

Pronunciation	Usage
nil	Used to report sports scores
nought	Not used in the USA

Examples

Written	Said
3.04+2.02=5.06	Three point zero four plus two point zero two makes five point zero six.
There is a 0% chance of rain.	There is a zero percent chance of rain.
The temperature is -20° C.	The temperature is twenty degrees below zero.
You can reach me at 0171 390 1062.	You can reach me at zero one seven one, three nine zero, one zero six two
I live at 4604 Smith Street.	I live at forty-six o four Smith Street
He became king in 1409.	He became king in fourteen o nine.

Written	**Said**
I waited until 4:05.	I waited until four o five.
The score was 4-0.	The score was four nil.

2) symbols

If we write the words "adding 4 to 2 gives 6" repeatedly, it might complicate things. These words also occupy more space and take time to write. Instead, we can save time and space by using symbols.

Basic math symbols

Symbol	Symbol Name	Meaning / definition	Example
=	equals sign	equality	5 = 2+3 5 is equal to 2+3
≠	not equal sign	inequality	$5 \neq 4$ 5 is not equal to 4
≈	approximately equal	approximation	$sin(0.01) \approx 0.01$, $x \approx y$ means x is approximately equal to y
>	strict inequality	greater than	5 > 4 5 is greater than 4
<	strict inequality	less than	4 < 5 4 is less than 5
≥	inequality	greater than or equal to	$5 \geq 4$, $x \geq y$ means x is greater than or equal to y

Symbol	Symbol Name	Meaning / definition	Example
≤	inequality	less than or equal to	4 ≤ 5, x ≤ y means x is less than or equal to y
( )	parentheses	calculate expression inside first	2 × (3+5) = 16
[ ]	brackets	calculate expression inside first	[(1+2)×(1+5)] = 18
+	plus sign	addition	1 + 1 = 2
−	minus sign	subtraction	2 − 1 = 1
±	plus - minus	both plus and minus operations	3 ± 5 = 8 or -2
±	minus - plus	both minus and plus operations	3 ∓ 5 = -2 or 8
*	asterisk	multiplication	2 * 3 = 6
×	times sign	multiplication	2 × 3 = 6
·	multiplication dot	multiplication	2 · 3 = 6
÷	division sign / obelus	division	6 ÷ 2 = 3
/	division slash	division	6 / 2 = 3
—	horizontal line	division / fraction	$\frac{6}{2} = 3$
mod	modulo	remainder calculation	7 mod 2 = 1
.	period	decimal point, decimal separator	2.56 = 2+56/100
a^b	power	exponent	$2^3 = 8$
a^b	caret	exponent	2 ^ 3 = 8
√a	square root	√a · √a = a	√9 = ±3
$\sqrt[3]{a}$	cube root	$\sqrt[3]{a} \cdot \sqrt[3]{a} \cdot \sqrt[3]{a} = a$	$\sqrt[3]{8} = 2$

Symbol	Symbol Name	Meaning / definition	Example
$\sqrt[4]{a}$	fourth root	$\sqrt[4]{a} \cdot \sqrt[4]{a} \cdot \sqrt[4]{a} \cdot \sqrt[4]{a} = a$	$\sqrt[4]{16} = \pm 2$
$\sqrt[n]{a}$	n-th root (radical)		for n=3, $\sqrt[n]{8} = 2$
%	percent	1% = 1/100	10% × 30 = 3
‰	per-mille	1‰ = 1/1000 = 0.1%	10‰ × 30 = 0.3
ppm	per-million	1ppm = 1/1000000	10ppm × 30 = 0.0003
ppb	per-billion	1ppb = 1/1000000000	10ppb × 30 = 3×10^{-7}
ppt	per-trillion	1ppt = 10^{-12}	10ppt × 30 = 3×10^{-10}

Geometry symbols

Symbol	Symbol Name	Meaning / definition	Example
∠	angle	formed by two rays	∠ABC = 30°
∟	right angle	= 90°	α = 90°
°	degree	1 turn = 360°	α = 60°
deg	degree	1 turn = 360deg	α = 60deg
′	prime	arcminute, 1° = 60′	α = 60°59′
″	double prime	arcsecond, 1′ = 60″	α = 60°59′59″
$\overleftrightarrow{AB}$	line	infinite line	
$\overline{AB}$	line segment	line from point A to point B	
$\overrightarrow{AB}$	ray	line that start from point A	
$\overset{\frown}{AB}$	arc	arc from point A to point B	$\overset{\frown}{AB}$ = 60°
⊥	perpendicular	perpendicular lines (90° angle)	$\overline{AC} \perp \overline{BC}$
∥	parallel	parallel lines	$\overline{AB} \parallel \overline{CD}$

Symbol	Symbol Name	Meaning / definition	Example
≅	congruent to	equivalence of geometric shapes and size	ΔABC≅ ΔXYZ
~	similarity	same shapes, not same size	ΔABC~ ΔXYZ
Δ	triangle	triangle shape	ΔABC≅ ΔBCD
|x-y|	distance	distance between points x and y	| x-y | = 5
π	pi constant	π = 3.141592654... is the ratio between the circumference and diameter of a circle	c = π·d = 2·π·r
rad	radians	radians angle unit	360° = 2π rad
c	radians	radians angle unit	$360° = 2\pi^{c}$
grad	gradians / gons	grads angle unit	360° = 400 grad
g	gradians / gons	grads angle unit	$360° = 400^{g}$

Algebra symbols

Symbol	Symbol Name	Meaning / definition	Example
x	x variable	unknown value to find	when $2x = 4$, then $x = 2$
≡	equivalence	identical to	
≜	equal by definition	equal by definition	
:=	equal by definition	equal by definition	
~	approximately equal	weak approximation	11 ~ 10
≈	approximately equal	approximation	$sin(0.01) \approx 0.01$

Symbol	Symbol Name	Meaning / definition	Example
$\propto$	proportional to	proportional to	$y \propto x$ when $y = kx$, k constant
∞	lemniscate	infinity symbol	
$\ll$	much less than	much less than	$1 \ll 1000000$
$\gg$	much greater than	much greater than	$1000000 \gg 1$
( )	parentheses	calculate expression inside first	2 * (3+5) = 16
[ ]	brackets	calculate expression inside first	[(1+2)*(1+5)] = 18
{ }	braces	set	
$\lfloor x \rfloor$	floor brackets	rounds number to lower integer	$\lfloor 4.3 \rfloor = 4$
$\lceil x \rceil$	ceiling brackets	rounds number to upper integer	$\lceil 4.3 \rceil = 5$
x!	exclamation mark	factorial	4! = 1*2*3*4 = 24
$\| x \|$	vertical bars	absolute value	$\| -5 \| = 5$
f (x)	function of x	maps values of x to f(x)	f (x) = 3x+5
$(f \circ g)$	function composition	$(f \circ g)(x) = f(g(x))$	$f(x)=3x, g(x)=x-1 \Rightarrow (f \circ g)(x)=3(x-1)$
(a,b)	open interval	$(a,b) = \{x \mid a < x < b\}$	$x \in (2,6)$
[a,b]	closed interval	$[a,b] = \{x \mid a \le x \le b\}$	$x \in [2,6]$
Δ	delta	change / difference	$\Delta t = t_1 - t_0$
Δ	discriminant	$\Delta = b2 - 4ac$	
$\sum$	sigma	summation - sum of all values in range of series	$\sum x_i = x_1+x_2+...+x_n$
$\sum\sum$	sigma	double summation	$\sum_{j=1}^{2}\sum_{i=1}^{8} x_{i,j} = \sum_{i=1}^{8} x_{i,1} + \sum_{i=1}^{8} x_{i,2}$

Symbol	Symbol Name	Meaning / definition	Example
∏	capital pi	product - product of all values in range of series	$\prod x_i = x_1 \cdot x_2 \cdot \ldots \cdot x_n$
e	e constant / Euler's number	e = 2.718281828...	$e = \lim (1+1/x)^x$, $x \to \infty$

γ	Euler-Mascheroni constant	γ = 0.5772156649...	
φ	golden ratio	golden ratio constant	
π	pi constant	$\pi = 3.141592654...$ is the ratio between the circumference and diameter of a circle	$c = \pi \cdot d = 2 \cdot \pi \cdot r$

3) Equation

Elastic Deformation

- STRESS–STRAIN BEHAVIOR

The degree to which a structure deforms or strains depends on the magnitude of an imposed stress. For most metals that are stressed in tension and at relatively low levels, stress and strain are proportional to each other through the relationship

$$\sigma = E\,\varepsilon \qquad (1)$$

This is known as *Hooke's law*, and the constant of proportionality *E* (GPa or psi)[6] is the **modulus of elasticity,** or *Young's modulus*. For most typical metals, the magnitude of this modulus ranges between 45 GPa (6.5×10^6 psi), for magnesium, and 407 GPa (59×10^6 psi), for tungsten. The moduli of elasticity are slightly higher for ceramic materials, which range between about 70 and 500 GPa (10×10^6 and 70×10^6 psi). Polymers have modulus values that are smaller than both metals and ceramics and lie in the range 0.007 to 4 GPa (10^3 to 0.6×10^6 psi).

Deformation in which stress and strain are proportional is called **elastic deformation;** a plot of stress (ordinate) versus strain (abscissa) results in a linear relationship, as shown in Figure1 The slope of this linear segment corresponds to the modulus of elasticity *E*. This modulus may be thought of as stiffness, or a material's resistance to elastic deformation. The greater the modulus, the stiffer is the material, or the smaller is the elastic strain that results from the application of a given stress. The modulus is an important design parameter used for computing elastic deflections.

Elastic deformation is nonpermanent, which means that when the applied load is released, the piece returns to its original shape. As shown in the stress–strain.

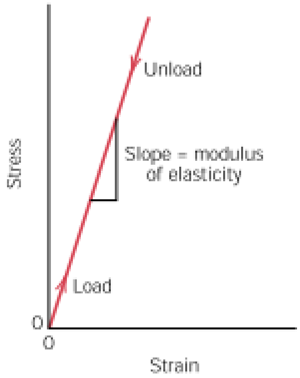


Figure 1 Schematic stress–strain diagram showing linear elastic deformation for loading and unloading cycles.

Differences in modulus values between metals, ceramics, and polymers are a direct consequence of the different types of atomic bonding that exist for the three materials types. Furthermore, with increasing temperature, the modulus of elasticity diminishes for all but some of the rubber materials.

As would be expected, the imposition of compressive, shear, or torsional stresses also evokes elastic behavior. The stress–strain characteristics at low stress levels are virtually the same for both tensile and compressive situations, to include the magnitude of the modulus of elasticity. Shear stress and strain are proportional to each other through the expression

$$\tau = G\,\gamma \qquad (2)$$

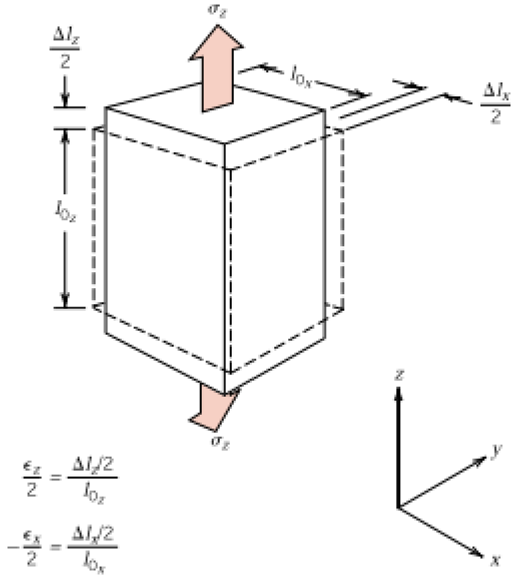


Figure 2 Axial (z) elongation (positive strain) and lateral (x and y) contractions (negative strains) in response to an imposed tensile stress. Solid lines represent dimensions after stress application; dashed lines, before.

There will be constrictions in the lateral (*x* and *y*) directions perpendicular to the applied stress; from these contractions, the compressive strains ε_x and ε_y may be determined. If the applied stress is uniaxial (only in the *z* direction) and the material is isotropic, then $\varepsilon_x = \varepsilon_y$.

A parameter termed **Poisson's ratio** ν is defined as the ratio of the lateral and axial strains, or

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z} \qquad (3)$$

EXAMPLE PROBLEM 1

A piece of copper originally 305 mm long is pulled in tension with a stress of 276 MPa (40,000 psi). If the deformation is entirely elastic, what will be the resultant elongation?

Solution

Since the deformation is elastic, strain is dependent on stress according to Equation 1. Furthermore, the elongation Δl is related to the original length l_0 through Equation

$$\varepsilon = \frac{l_i - l_0}{l_0} = \frac{\Delta l}{l_0}$$

Combining these two expressions and solving for Δl yields

$$\sigma = \epsilon E = \left(\frac{\Delta l}{l_0}\right) E$$

$$\Delta l = \frac{\sigma l_0}{E}$$

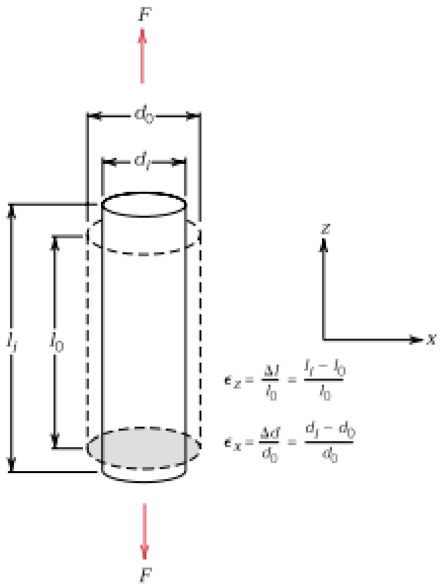
The values of σ and l_0 are given as 276 MPa and 305 mm, respectively, and the magnitude of *E* for copper is 110 GPa (16×10^6 psi). Elongation is obtained by substitution into the expression above as

$$\Delta l = \frac{(276\,MPa)(305mm)}{110\times10^3 MPa} = 0.77mm$$

EXAMPLE PROBLEM 2

A tensile stress is to be applied along the long axis of a cylindrical brass rod that has a diameter of 10 mm. Determine the magnitude of the load required to produce a 2.5×10^{-3}mm; change in diameter if the deformation is entirely elastic.

Solution
This deformation situation is represented in the accompanying drawing.



When the force *F* is applied, the specimen will elongate in the *z* direction and at the same time experience a reduction in diameter, Δd, of 2.5×10^{-3} mm in the *x* direction. For the strain in the *x* direction,

$$\epsilon_x = \frac{\Delta d}{d_0} = \frac{-2.5 \times 10^{-3} mm}{10mm} = 2.5 \times 10^{-4}$$

which is negative since the diameter is reduced.

It next becomes necessary to calculate the strain in the *z* direction using Equation. The value for Poisson's ratio for brass is 0.34, and thus

$$\epsilon_z = -\frac{\epsilon_x}{\nu} = -\frac{(-2.5 \times 10^{-4})}{0.34} = 7.35 \times 10^{-4}$$

The applied stress may now be computed using Equation 1 and the modulus of elasticity, given as 97 GPa (14×10^6 psi), as

$$\sigma = \epsilon_z E = (7.35 \times 10^{-4})(97 \times 10^3 MPa) = 71.3\ MPa$$