Corrigé TD N°01 Maths3 (2021 / 2022)

Exercice N°01

$$\int (2x^3 - 3x + 1)dx = \frac{2}{4}x^4 - \frac{3}{2}x^2 + x + c\,, c \in \mathbb{R}$$

$$\int (3x^2+4)^3 x\,dx = \frac{1}{6}\int (3x^2+4)^3(6x)\,dx = \frac{1}{6}\int (u(x))^3 \times u'(x)dx \quad \text{avec} \quad u(x) = 3x^2+4$$

$$= \frac{1}{6}\left[\frac{1}{4}\,(u(x))^4\right] + c = \frac{1}{24}\,(3x^2+4)^4 + c\,.$$

$\int \ln(1+x)dx = ?$ intégration par parties

$$\begin{cases} u(x) = \ln(1+x) \\ v'(x) = 1 \end{cases} \Longrightarrow \begin{cases} u'(x) = \dfrac{1}{1+x} \\ v(x) = x \end{cases}$$

$$\int \ln(1+x)dx = x\ln(1+x) - \int \frac{x}{1+x}dx$$

$$= x\ln(1+x) - \int \frac{1+x-1}{1+x}dx$$

$$= x\ln(1+x) - \int \left(1 - \frac{1}{1+x}\right)dx$$

$$= x\ln(1+x) - x + \ln(1+x) + c$$

$$= (1+x)\ln(1+x) - x + c \quad , c \in \mathbb{R}\,.$$

$$\int \frac{x}{1+x^4}dx\,, \text{posons } t = x^2, \frac{}{dx} = 2x \Longrightarrow dt = 2x\,dx.$$

$$\text{Alors} \int \frac{x}{1+x^4}dx = \frac{1}{2}\int \frac{2x}{1+x^4}dx$$

$$= \frac{1}{2}\int \frac{1}{1+t^2}dt$$

$$= \frac{1}{2}\arctan(t) + c$$

$$= \frac{1}{2}\arctan(x^2) + c\,, \quad c \in \mathbb{R}.$$

$$\int \frac{x}{(x^2+1)^6}dx$$

$$= \tfrac{1}{2}\int \frac{2x}{(x^2+1)^6}dx$$

on choisit $u(x) = x^2+1$ alors $u'(x) = 2x$

donc $\quad = \tfrac{1}{2}\int \frac{u'(x)}{(u(x))^6}dx = \tfrac{1}{2}\int_5 \frac{1}{u^6}du = \tfrac{1}{-10}u^{-5}+c, c\in\mathbb{R}$

$$= \tfrac{1}{-10}(x^2+1)^{-5}+c, c\in\mathbb{R}$$

$$\int \frac{x^2}{\sqrt{x^3+1}}dx,$$

$$= \int \frac{x^2}{\sqrt{x^3+1}}dx = \tfrac{1}{3}\int \frac{3x^2}{\sqrt{x^3+1}}dx$$

on choisit $u(x) = x^3+1$ alors $u'(x) = 3x^2$

$$= \tfrac{1}{3}\int \frac{u'(x)}{\sqrt{u(x)}}dx = \tfrac{1}{3}\int \frac{1}{\sqrt{u}}du = \tfrac{2}{3}u^{\frac{1}{2}}+c, c\in\mathbb{R}$$

$$= \tfrac{2}{3}\sqrt{x^3+1}+c, c\in\mathbb{R}$$

nous utilisons meme methode pour trouver:

$$\int \frac{\cos x}{\sin x+1}dx = \ln|\sin x+1|+c, c\in\mathbb{R}$$

Exercice n°02

$$\int_{-1}^{1} \frac{1}{1+x^2}dx = [\arctan(x)+c]_{-1}^{1} \quad , \qquad c\in\mathbb{R}$$

$$= \arctan(1)+c-(\arctan(-1)+c)$$

$$= \arctan(1)-\arctan(-1) = \frac{\pi}{4}-\left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$$

$$\int_{-1}^{1} \frac{2x-5}{x^2-5x+6}dx = [\ln|x^2-5x+6|]_{-1}^{1} = \ln 2-\ln 12$$

$$= \ln 2-\ln 3-2\ln 2 \qquad = -\ln 2-\ln 3.$$

$$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{1}{1+3x^2}dx = \int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{1}{1+\left(\sqrt{3}x\right)^2}dx$$

On pose $t = \sqrt{3}x \Longrightarrow dx = \frac{1}{\sqrt{3}}dt$

si $x \longrightarrow \frac{1}{\sqrt{3}}$, $t \longrightarrow 1$ et si $x \longrightarrow -\frac{1}{\sqrt{3}}$, $t \longrightarrow -1.$

Alors

$$\int_{-1/\sqrt{3}}^{1/\sqrt{3}} \frac{1}{1+3x^2}dx = \frac{1}{\sqrt{3}}\int_{-1}^{1} \frac{1}{1+t^2}dt = \frac{1}{\sqrt{3}}\times\frac{\pi}{2} = \frac{\pi}{2\sqrt{3}}$$

Exercice N°03

$1) I = \left[\int_1^3 x dx\right] \times \left[\int_1^5 \frac{1}{y} dy\right] = 4\ln 5.$

2) $I_1 = \iint_D (x^2 + y).dx.dy \quad ; \qquad D = \{(x, y) \in \mathbb{R}^2, 0 \le x \le 1; \; x - 1 \le y \le 1 - x\}$

En utilisant le théorème de fubinni

$$\int_0^1 (\int_{x-1}^{1-x} (x^2 + y)dy)dx = \int_0^1 \left[x^2.y + \frac{1}{2}y^2\right]_{x-1}^{1-x} dx$$

$$= \int_0^1 [x^2(1-x) + \frac{1}{2}(1-x)^2 - x^2(x-1) - \frac{1}{2}(x-1)^2)dx \; = \frac{1}{6}$$

3) $I_2 = \iint_D \frac{x^2 + y^2}{\sqrt{x^2 + y^2}} dx.dy \quad ; \qquad D = \{(x, y) \in \mathbb{R}^2, x^2 + y^2 \le 1\}$

Changement de variable :

$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ dx.dy = rdrd\theta \\ 0 \le r \le 1 \;\; ; \;\; 0 \le \theta \le 2\pi \end{cases}$$

On obtient :

$$I_2 = \int_0^1 \int_0^{2\pi} \frac{r^2(\cos\theta^2 + \sin\theta^2)}{\sqrt{r^2(\cos\theta^2 + \sin\theta^2)}} rdrd\theta = \int_0^1 \int_0^{2\pi} \frac{r^2}{\sqrt{r^2}} rdrd\theta$$

$$= \int_0^1 r^3 dr \int_0^{2\pi} 1\, d\theta = \left|\frac{1}{4}r^4\right|_0^1 . |\theta|_0^{2\pi} = \frac{\pi}{2}$$

Exercice N°04

1) $I = \iint_D (yxz)\, dxdydz$,où $D = [1, 3]^3$

$$I = \left[\int_1^3 x dx\right] \times \left[\int_1^3 y dy\right] \times \left[\int_1^3 z dz\right] = 64$$

2) $I = \iint_D (x + y + 2z)\, dxdydz$,où $D = \{(x, y) \in \mathbb{R}^2; (x, y) \in [5, 7]^2, y \le z \le x\}$

$$I = \iint_D (x + y + 2z)\, dxdydz = \iint_{[5,7]^2} \left[\int_y^x (x + y + 2z)\, dz\right] dxdy$$

$$= \iint_{[5,7]^2} 2\left(x^2 - y^2\right) dxdy$$

$$= 2\int_5^7 \left[\int_5^7 \left(x^2 - y^2\right) dx\right] dy$$

$$=$$