



(1h)

Elément de correction de la série de TD N°02 (électrostatique)

EX01:

dg sur le disque
Crée au point M
un élément de champ

$$d\vec{E} = \frac{K \cdot dq}{d^2} \vec{u}$$

pour des raisons de symétrie

$$dE_T = dE_3 =$$

$$dE_3 = dE \cdot \cos \alpha$$

$S = \pi r^2$ surface d'un disque

$ds = 2\pi r dr$ élément de surface

$$\sigma = \frac{dq}{ds} \Rightarrow dq = \sigma \cdot ds$$

$$dq = \sigma \cdot 2\pi \cdot r \cdot dr$$

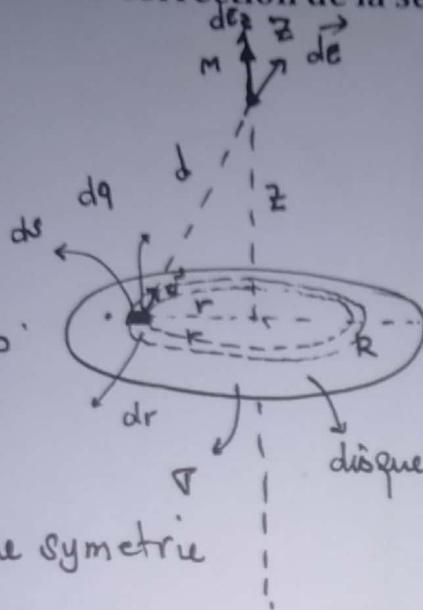
$$d^2 = r^2 + z^2 \Rightarrow d = \sqrt{r^2 + z^2}$$

$$\cos \alpha = \frac{z}{d} = \frac{z}{\sqrt{r^2 + z^2}}$$

$$dE_3 = dE \cdot \cos \alpha$$

$$= \frac{K \cdot dq}{d^2} \cdot \frac{z}{d} = \frac{K \cdot 4\pi \cdot \sigma \cdot r \cdot dr}{(r^2 + z^2)^{3/2}}$$

$$K = \frac{1}{4\pi\epsilon_0}$$



$$dE_z = \frac{\sigma z}{2\epsilon_0} \frac{r \cdot dr}{(r^2 + z^2)^{3/2}}$$

$$\int E_z = \frac{\sigma z}{2\epsilon_0} \int_0^R \frac{r \cdot dr}{(r^2 + z^2)^{3/2}}$$

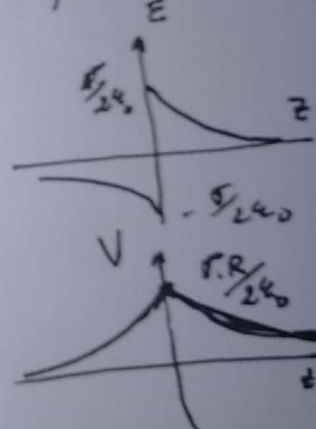
$$E_z = \frac{\sigma z}{2\epsilon_0} \left[\frac{1}{(r^2 + z^2)^{1/2}} \right]_0^R$$

$$E_z = \frac{\sigma z}{2\epsilon_0} \left[\frac{1}{|z|} - \frac{1}{(R^2 + z^2)^{1/2}} \right]$$

b) le champ créé par un plan

$$R \rightarrow +\infty$$

$$E_z = \frac{\sigma}{2\epsilon_0}$$



c) le potentiel (V)

au point M. $z > 0$

$$E_z = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{z}{(R^2 + z^2)^{1/2}} \right]$$

$$\vec{E} = -\text{grad } V$$

$$\vec{E} = -\frac{\partial V}{\partial x} \vec{i} - \frac{\partial V}{\partial y} \vec{j} - \frac{\partial V}{\partial z} \vec{k}$$

$$\vec{E} = -\frac{\partial V}{\partial z} \vec{k}$$

$$E = -\frac{\partial V}{\partial z} \Rightarrow \partial V = -E \cdot dz$$

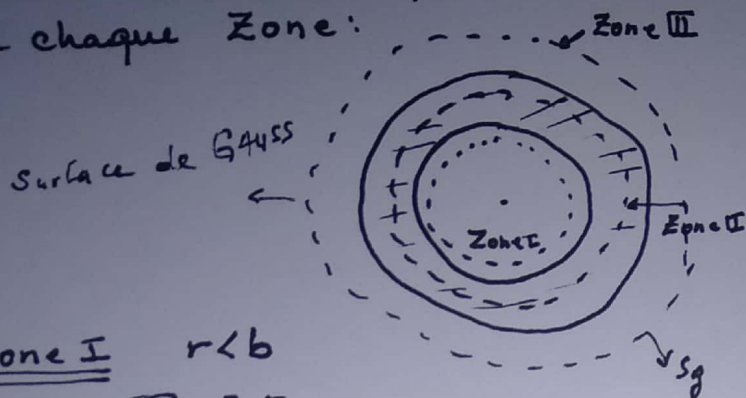
$$V = -\int E \cdot dz$$

$$V = \frac{\sigma}{2\epsilon_0} \left[(R^2 + z^2)^{1/2} - z \right] + C$$

Serie TD N=2

EX 2

① La charge électrique à l'intérieur de chaque Zone:



Zone I $r < b$
 $Q_{int} = 0$

Zone II $b < r < a$
 $V = \frac{4}{3}\pi r^3 \Rightarrow dV = 4\pi r^2 \cdot dr$
 $\rho = \frac{dq}{dV} \Rightarrow dq = \rho \cdot dV$

$dq = \rho \cdot dV$
 $dq = \frac{\rho_0 \cdot r}{4a} \cdot 4\pi r^2 \cdot dr$

$dq = \frac{\pi \cdot \rho_0}{a} \cdot r^3 \cdot dr$

$Q_{II} = \int dq = \frac{\pi \rho_0}{a} \int_b^a r^3 \cdot dr$

$Q_{II} = \frac{\pi \rho_0}{4a} [r^4 - b^4]$

Zone III: $r > a$
 $Q_{III} = \frac{\pi \rho_0}{4a} [a^4 - b^4]$

② la relation du Théorème de GAUSS:
 $\Phi = \oint_S \vec{E} \cdot d\vec{S} = \frac{Q_{int}}{\epsilon_0}$

~~Zone I~~

③ le champ et le potentiel Electrostatique

Zone-I

$\Phi_I = \oint_S \vec{E}_I \cdot d\vec{S}_I = \frac{Q_I}{\epsilon_0}$ $S_I = 4\pi r^2$
 $= E_I \cdot S_I = 0 \Rightarrow E_I = 0 \text{ V/m}$

$\vec{E} = -\text{grad} V$

$\vec{E} = -\frac{dV}{dr} \vec{u}_r$

$dV = -E \cdot dr$

$V_I = \text{cte}$

Zone-II

$\Phi_{II} = E_{II} \cdot S_{II} = \frac{Q_{II}}{\epsilon_0}$

$E_{II} \cdot 4\pi r^2 = \frac{\pi \rho_0}{4a \epsilon_0} [r^4 - b^4]$

$E_{II} = \frac{\rho_0}{16a \epsilon_0} \left[r^2 - \frac{b^4}{r^2} \right]$

$dV = -E \cdot dr$

$V_{II} = \frac{-\rho_0}{16a \epsilon_0} \left[\frac{1}{3} r^3 + \frac{b^4}{r} \right] + C$

Zone-III

$\Phi_{III} = E_{III} \cdot S_{III} = \frac{Q_{III}}{\epsilon_0}$

$E_{III} \cdot 4\pi r^2 = \frac{\rho_0 \pi}{4a \epsilon_0} [a^4 - b^4]$

$E_{III} = \frac{\rho_0}{16a \epsilon_0} [a^4 - b^4] \cdot \frac{1}{r^2}$

$dV = -E \cdot dr$

$V_{III} = \int dV = -\int \frac{\rho_0}{16a \epsilon_0} [a^4 - b^4] \cdot \frac{1}{r^2} \cdot dr$

$V_{III} = \frac{\rho_0}{16a \epsilon_0} [a^4 - b^4] \cdot \frac{1}{r} + \text{cte}$