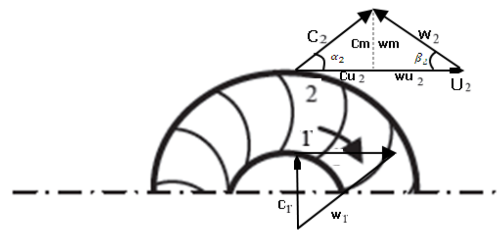
Corrigé de la fiche des travaux dirigés N°1

Solution 1 :

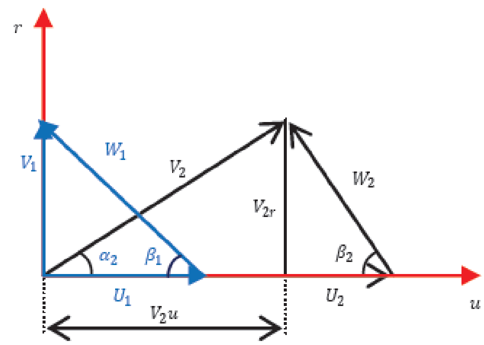
1. **les triangles des vitesses et leurs éléments :** $(U_1, U_2, w_1, w_2, C_2, \beta_1, \beta_2, \alpha_2)$

$\boldsymbol{Q_v = 8.5\frac{l}{s} = 8.5 \times 10^{-3} m^3/s}$, $N = 3000\frac{tr}{min}$, $D_1 = 74\ mm$, $D_2 = 140\ mm$, $C_1 = C_{2m} = 3\frac{m}{s}$, $W = 310\frac{J}{kg}$.



$$U_1 = \frac{\pi N D_1}{60} = \frac{\pi * 3000 * 0.074}{60} = 11.62\frac{m}{s}$$

$U_2 = \frac{\pi N D_2}{60} = \frac{\pi * 3000 * 0.14}{60} = 21.98\frac{m}{s}$



$$w_1 = \sqrt{U_1^2 + C_1^2} = \sqrt{11.62^2 + 3^2} = 12\frac{m}{s}$$

On a le travail: $W = U_2 * C_{2u} - U_1 * C_{1u}$ $(theoreme\ d'Euler)$

Entrée radial : $C_{1u} = 0$, $W = U_2 * C_{2u} \Leftrightarrow C_{2u} = \frac{W}{U_2} = \frac{310}{21.98}$

$$C_{2u} = 14.10\frac{m}{s}$$

$$C_2 = \sqrt{C_{2u}^2 + C_{2m}^2} = \sqrt{14.1^2 + 3^2} = 14.42\frac{m}{s}$$

$$w_2 = \sqrt{(U_2 - C_{2u})^2 + C_{2m}^2} = \sqrt{(21.98 - 14.1)^2 + 3^2} = 8.43\frac{m}{s}$$

$$\beta_1 = arcsin\left(\frac{C_1}{U_1}\right) = arcsin\left(\frac{3}{11.62}\right) = 14.48°$$

$$\beta_2 = arcsin\left(\frac{C_{2m}}{U_2 - C_{2u}}\right) = arcsin\left(\frac{3}{21.91 - 14.1}\right) = 20.84°$$

$$\alpha_2 = arcsin\left(\frac{C_{2m}}{C_{2u}}\right) = arcsin\left(\frac{3}{14.1}\right) = 12.01°$$

2. **Calcul des largeurs b_1 et b_2**

$$Q_v = \pi D_1 b_1 C_1 = \pi D_2 b_2 \boldsymbol{C_{2m}}$$

$$b_1 = 12.2\ mm$$

$$b_2 = 6.4\ mm$$

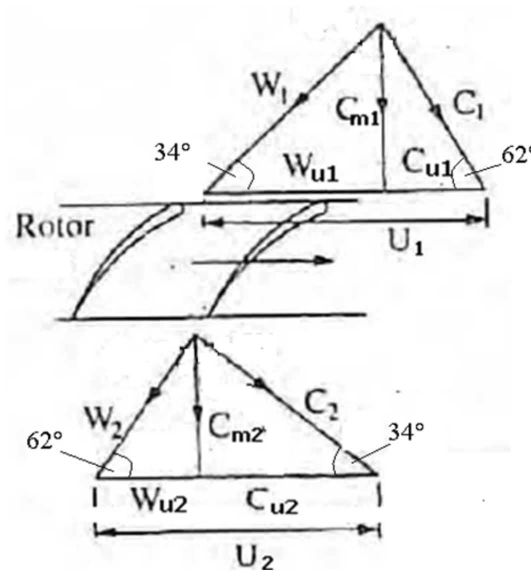
Solution 2 :

1. **La vitesse axiale (C_m) :**

$$U_1 = C_{u,1} + W_{u,1}$$

$$\tan\alpha_1 = \frac{C_{1,m}}{C_{1,u}} \leftrightarrow C_{1,u} = \frac{C_{1,m}}{\tan 62°}$$

$$\tan\beta_1 = \frac{W_{1,m}}{W_{1,u}} \leftrightarrow W_{1,u} = \frac{W_{1,m}}{\tan 34°} = \frac{C_{1,m}}{\tan 34°}$$



$$U_1 = \frac{C_{1,m}}{\tan 62°} + \frac{C_{1,m}}{\tan 34°}$$

$$U_1 = C_{1,m}\left(\frac{1}{\tan 62°} + \frac{1}{\tan 34°}\right)$$

$$C_{m,1} = \frac{U_1}{\frac{1}{\tan 62°} + \frac{1}{\tan 34°}}$$

$$U_1 = \frac{\pi DN}{60} = \frac{\pi * 0.95 * 5000}{60}$$

$$U_1 = 248.71\frac{m}{s}$$

$$C_{m,1} = \frac{248.70}{\frac{1}{\tan 62°} + \frac{1}{\tan 34°}}$$

$$C_{m,1} = 123.47\frac{m}{s}$$

2. **Le débit massique**

$$\dot{m} = \rho C_{m,1} A$$

$$\dot{m} = 1.2 * 123.47 * 1.14$$

$$\dot{m} = 20.94\frac{kg}{s}$$

3. **La puissance totale**

$$P_c = \dot{m}(U_2 * c_{2u} - U_1 * c_{1u})$$

$$C_{1,u} = \frac{C_{1,m}}{\tan 62°}$$

$$C_{2,u} = \frac{C_{2,m}}{\tan 34°}$$

$$C_{1,m} = C_{2,m}\ (machine\ axiale)$$

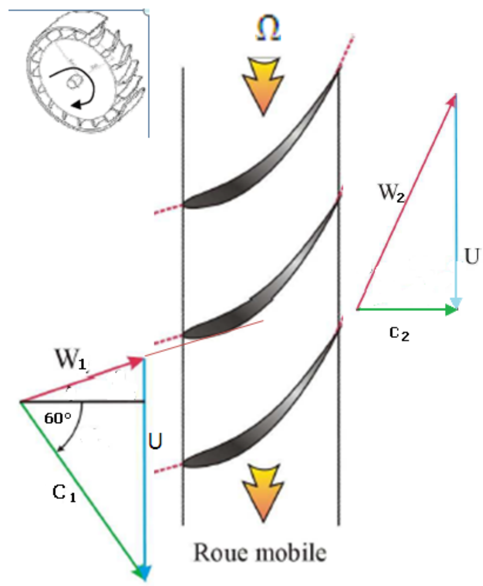
$$P_c = \dot{m} * U * C_{1,m}\left(\frac{1}{\tan 34°} - \frac{1}{\tan 62°}\right)$$

$$P_c = 20.94 * 248.71 * 123.47 * \left(\frac{1}{\tan 34°} - \frac{1}{\tan 62°}\right)$$

$$P_c = 611625.38\frac{J}{kg} = 611.625\frac{kJ}{kg}$$

Solution 4 :

1. **Les triangles des vitesses à l'entrée et à la sortie**



$$sin60° = \frac{Cu_1}{C_1} \leftrightarrow Cu_1 = C_1 * sin60°$$

$$Cu_1 = 658.17\frac{m}{s}$$

$$Cu_2 = 0\ (direction\ purement\ axiale)$$

2. **Le travail spécifique réel et la puissance réelle du rotor**

- **Le travail spécifique réel**

$$W = (Cu_1U_1 - Cu_2U_2)$$

$$U_1 = \Omega * r_1 = \frac{2\pi N}{60}r_1$$

$$U_1 = 502.65\frac{m}{s}$$

$$W = Cu_1 U_1 = 658.17 * 502.65$$

$$W = 330.829\ kJ/kg$$

- **La puissance spécifique réelle**

$$\dot{W}_{réel} = \dot{m}\, W = 1.2 * 330.829$$

$$\dot{W}_{réel} = 396.994\ \ kW$$

3. **Le travail spécifique théorique du rotor**

$$\eta_{is} = \frac{W_{réel}}{W_{theo}} \Leftrightarrow W_{theo} = \frac{W_{réel}}{\eta_{is}} = \frac{330.829}{0.85}$$

$$W_{theo} = 389.21\ kJ/kg$$

4. **L'enthalpie h_{02s} et la température T_{02s}**

$$W_{theo} = W_{isen} = (h_{01} - h_{02s}) \Leftrightarrow h_{02s} = h_{01} - W_{theo} = T_{01}Cp - W_{theo}$$

$$Cp = \frac{\gamma R}{\gamma - 1} = 1004.5\,\frac{J}{kg\ K}$$

$$h_{02s} = 1333 * 1004.5 - 389210$$

$$h_{02s} = 949788.5\,\frac{J}{kg}$$

$$h_{02s} = T_{02s}Cp \leftrightarrow T_{02s} = \frac{h_{02s}}{Cp} = \frac{949788.5}{1004.5}$$

$$T_{02s} = 945.53\ K$$

Solution 5 :

1.

La vitesse spécifique est :

$$\Omega = \frac{333 * \pi}{30} = 34.87\,\frac{\text{rad}}{\text{s}}$$

$$N_s = \frac{\Omega Q^{1/2}}{(gH)^{3/4}} = N_s = \frac{34.87 * (71)^{1/2}}{(9.81 * 543)^{3/4}}$$

$$N_s = 0.471\ rad$$

Le Diamètre spécifique est :

$$D_s = \frac{D(gH)^{\frac{1}{4}}}{Q^{\frac{1}{2}}} = \frac{4.31(9.81 * 543)^{\frac{1}{4}}}{(71)^{\frac{1}{2}}}$$

$$D_s = 4..36$$

La puissance hydraulique nette est :

$$P_{net} = \rho g Q H = 1000 * 9.81 * 71 * 543$$

$$P_{net} = 378.20\ 10^6\ W = 378.20\ \ MW$$

Le rendement de la turbine est :

$$\eta_{T,H} = \frac{P_{act}}{P_{net}} = \frac{350}{378.20} = 0.92$$

2.

Le débit volumique nécessaire :

$$Q = \left(\frac{D}{D_s}\right)^2 (gH)^{\frac{1}{2}} = \left(\frac{6}{4.36}\right)^2 (9.81 * 500)^{\frac{1}{2}}$$

$$Q = 132.63 \frac{m^3}{s}$$

La puissance actuel, P_{act}, est :

$$P_{act} = \eta_{T,H} * P_{net} = \eta_{T,H} * \rho g Q H$$

$$P_{act} = 0.92 * 9.81 * 1000 * 132.63 * 500$$

$$P_{act} = 598.52\ 10^6\ W = \ 598.52\ MW$$

La vitesse de rotation de la turbine est :

$$N_s = \frac{\Omega \mathrm{Q}^{1/2}}{(gH)^{3/4}} \Leftrightarrow \Omega = \frac{N_s (gH)^{3/4}}{\mathrm{Q}^{1/2}}$$

$$\Omega = \frac{0.473 * (9.81 * 500)^{3/4}}{132.63^{1/2}}$$

$$\Omega = 23.97\ \mathrm{rad/s}$$