Université Ahmed Zabana-Relizane **Sciences et Technologies** **2021/2022**

Première année ST -Maths 2

Fiche de TD 3

Exercice 1 : Calculer les primitives suivantes:

$(1.)\int(2x^3-3x+1)dx$, $(2.)\int(3x^2+4)^3x\,dx$, $(3.)\int\ln(1+x)dx$; $(4.)\int\frac{\sin(\ln x)}{x}dx$

Exercice 2 : Posons

$$A=\int e^{2x}\cos^2 x\ dx\quad et\ \ B=\int e^{2x}\sin^2 x\ dx$$

1) Calculer $A+B$

2) En appliquant la méthode d'intégration par parties deux fois, calculer $A-B$

3) Déduire $A\ \ et\ B$. **Indication** : $\cos 2x=\cos^2 x-\sin^2 x$

Correction de fiche TD 3

Solution de L'exercice 1:

$$\int (2x^3 - 3x + 1)dx = \frac{2}{4}x^4 - \frac{3}{2}x^2 + x + c\ , c \in \mathbb{R}$$

$$\int (3x^2+4)^3 x\, dx = \frac{1}{6}\int (3x^2+4)^3(6x)\, dx = \frac{1}{6}\int (u(x))^3 \times u'(x)dx \quad \text{avec} \quad u(x) = 3x^2+4$$

$$= \frac{1}{6}\left[\frac{1}{4}\,(u(x))^4\right] + c = \frac{1}{24}\,(3x^2+4)^4 + c\,.$$

$\int \ln(1+x)dx = ?$ intégration par parties

$$\begin{cases} u(x) = \ln(1+x) \\ v'(x) = 1 \end{cases} \Rightarrow \begin{cases} u'(x) = \dfrac{1}{1+x} \\ v(x) = x \end{cases}$$

$$\int \ln(1+x)dx = x\ln(1+x) - \int \frac{x}{1+x}dx$$

$$= x\ln(1+x) - \int \frac{1+x-1}{1+x}dx$$

$$= x\ln(1+x) - \int \left(1 - \frac{1}{1+x}\right)dx$$

$$= x\ln(1+x) - x + \ln(1+x) + c$$

$$= (1+x)\ln(1+x) - x + c \quad , c \in \mathbb{R}\,.$$

$\int \frac{\sin(\ln x)}{x}dx$, on pose $t = \ln x \Rightarrow \frac{dt}{dx} = \frac{1}{x} \Rightarrow dx = x\, dt.$

Alors $\int \frac{\sin(\ln x)}{x}dx = \int \sin t\ \ dt = -\cos t + c$

$$= -\cos(\ln x\,) + c\,, \qquad c \in \mathbb{R}.$$

Solution de L'exercice 2:

1. $A + B = \int e^{2x}(\cos^2 x + \sin^2 x)dx = \int e^{2x}dx = \frac{1}{2}e^{2x} + c_1$

2. $A - B = \int e^{2x}(\cos^2 x - \sin^2 x)dx = \int e^{2x}\cos 2x\, dx$

Posons $\begin{cases} u(x) = e^{2x} \\ v'(x) = \cos 2x \end{cases} \Rightarrow \begin{cases} u'(x) = 2e^{2x} \\ v(x) = \frac{1}{2}\sin 2x \end{cases}$

$$A - B = u(x) \cdot v(x) - \int u'(x) \cdot v(x) dx = \frac{1}{2} e^{2x} \sin 2x - \underbrace{\int e^{2x} \sin 2x \, dx}_{I} \qquad \dots\dots\dots\dots\dots (*)$$

Posons $\begin{cases} f(x) = e^{2x} \\ g'(x) = \sin 2x \end{cases} \Longrightarrow \begin{cases} f'(x) = 2e^{2x} \\ g(x) = -\frac{1}{2} \cos 2x \end{cases}$

$$I = f(x) \cdot g(x) - \int f'(x) \cdot g(x) dx = -\frac{1}{2} e^{2x} \cos 2x + \int e^{2x} \cos 2x \, dx$$

$$= -\frac{1}{2} e^{2x} \cos 2x + (A - B)$$

en remplace dans $(*)$

$$A - B = \frac{1}{2} e^{2x} \sin 2x + \frac{1}{2} e^{2x} \cos 2x - (A - B) \Leftrightarrow A - B = \frac{1}{4} e^{2x} \sin 2x + \frac{1}{4} e^{2x} \cos 2x$$

$$= \frac{1}{4} e^{2x} (\sin 2x + \cos 2x) + c_2$$

3. On a $\begin{cases} A + B = \frac{1}{2} e^{2x} + c_1 & \dots\dots\dots (1) \\ A - B = \frac{1}{4} e^{2x} (\sin 2x + \cos 2x) + c_2 & \dots\dots\dots (2) \end{cases}$

$$(1) + (2) \Leftrightarrow 2A = \frac{1}{2} e^{2x} + c_1 + \frac{1}{4} e^{2x} (\sin 2x + \cos 2x) + c_2$$

$$\Leftrightarrow 2A = \frac{1}{2} e^{2x} \left[1 + \frac{1}{2} \sin 2x + \frac{1}{2} \cos 2x\right] + c_1 + c_2 \ , \quad c_1 \text{ et } c_2 \in \mathbb{R}$$

$$\Leftrightarrow A = \frac{1}{4} e^{2x} \left[\frac{1}{2} (2 + \sin 2x + \cos 2x)\right] + \frac{c_1 + c_2}{2}$$

$$\Leftrightarrow A = \frac{1}{8} e^{2x} [2 + \sin 2x + \cos 2x] + c \ , \quad c = \frac{c_1 + c_2}{2} \in \mathbb{R}.$$

$$(1) \Longrightarrow B = \frac{1}{2} e^{2x} + c_1 - A = \frac{1}{2} e^{2x} + c_1 - \frac{1}{8} e^{2x} [2 + sin\, 2x + cos\, 2x] - c$$

$$B = \frac{1}{8} e^{2x} [4 - 2 - \sin 2x - \cos 2x] + c_1 - c$$

$$B = \frac{1}{8} e^{2x} [2 - \sin 2x - \cos 2x] + c' \ , \ \ c' = \frac{c_1 - c_2}{2} \in \mathbb{R}.$$